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A NOTE ON USING THE INTEGRATED FORM OF ARIMA FORECASTS

BY

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A Note on using the Integrated form of ARIMA forecasts

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ABSTRACT

Forecasts of ARIMA processes are generally made using the Difference Equation form. This is the approach favoured by Box and Jenkins and most subsequent authors. The purpose of this note is to emphasise that the Integrated Form of the forecast enjoys some important advantages derived from its explicit use of the Eventual Forecast Function (EFF). A brief review of the procedures for obtaining all the necessary components of the Integrated Form is given, and a new and direct method is derived for evaluating the extra coefficients necessary when the EFF does not yield the forecasts for all lead times.

In their book, Box and Jenkins (1970) strongly recommend that forecasts of ARIMA processes be made using the Difference Equation form because it is the simplest approach. All subsequent textbooks have endorsed this view to the extent that very few of them consider forecasting in the Integrated Form in any detail. A notable exception to this is the recent book by Abraham and Ledolter (1983) which contains some useful detailed discussion of the role of the Eventual Forecast Function (EFF) in generating forecasts. The purpose of this note is to indicate that the Integrated Form does have value and is worthy of consideration. We begin with a brief review of the derivation of the necessary components for this form of the forecast.

THE INTEGRATED FORM OF THE FORECAST

Suppose $\{X_t\}$ is an ARIMA process satisfying $\alpha(B)X_t = \beta(B)a_t$ where B is the Backshift Operator which is defined by $B^k X_t = X_{t-k}$, and where $\alpha(B) = \sum_{k=0}^{p} \alpha_k B^k$, $\beta(B) = \sum_{k=0}^{q} \beta_k B^k$, $\alpha_0 = \beta_0 = 1$ and all seasonal and difference terms are included in the α- and β- operators. For lead times $T > q$, $\sum_{k=0}^{p} \alpha_k \hat{X}_t(T-k) = 0$, where $\hat{X}_t(i)$ is the forecast made at time t of X_{t+i} and takes the value X_{t+i} if $i \leq 0$. The solution of this difference equation is the EFF. It can be written as

$$\hat{X}_t(T) = \sum_{k=1}^{p} b_k^t f_k(T) \qquad (1)$$

where $f_k(T)$ are deterministic functions of T and may include polynomials, exponentials, sinusoids and products of these. They may also be dummy variables generating a seasonal pattern.

As a representation of the T-step ahead forecast (1) above is valid for only $T > M = q - p$. If $M > 0$, then for lead times $T = 1,2,\ldots,M$

$$\hat{X}_t(T) = \sum_{k=1}^{p} b_k^t f_k(T) + \sum_{j=0}^{M-T} d_{T,j} e_{t-j} \qquad (2)$$

where $\{e_t\}$ is the sequence of one-step ahead forecast errors, i.e. $e_t = X_t - \hat{X}_{t-1}(1)$.

Box and Jenkins (1970) show that the current value of $\underline{b}_t = (b_1^t, b_2^t, \ldots b_p^t)'$ may be obtained from $\underline{b}_{t-1}$ via a linear equation $\underline{b}_t = L\underline{b}_{t-1} + \underline{h}e_t$. The matrix L effects the changes in the coefficients in revising the time origin from $(t-1)$ to t , and can be obtained as $L = F_M^{-1} F_{M+1}$, where F_M is the $(p \times p)$ matrix with (i,j)th element $f_j(M+i)$. The vector $\underline{h} = F_M^{-1} \underline{\psi}_M$, where $\underline{\psi}_M = (\psi_{M+1}, \psi_{M+2}, \ldots, \psi_{M+p})'$ and ψ_k is the coefficient of B^k in $\psi(B) = \beta(B)/\alpha(B)$, the usual moving-average representation of the ARIMA process.

We may note that the revision matrix L is block diagonal. Each block corresponds to a real (repeated) linear factor or conjugate pairs of complex factors in $\alpha(B)$. The effect of this structure is that individual components can be monitored, revised and projected independently of the others. Thus, for example, any linear trend and seasonal factors can be obtained at each time t given only their values at time $(t-1)$ and the latest forecast error e_t . Details of the derivation of the revision equations for the linear trend and seasonally differenced ARIMA model forecasts are given in McKenzie (1984b). They can be obtained without direct evaluation and inversion of the matrix F_M.

If we wish to generate not merely individual components of the EFF but the forecast itself using the Integrated form, we must use (1) or (2) above. If $M > 0$, it is necessary to obtain $\{\alpha_{T,i} : i = 0,1,\ldots,M-T\ ;\ T = 1,2,\ldots,M\}$. This problem is referred to only briefly by Box and Jenkins and appears to be a messy problem in algebra. However, we present here a general solution relating the α-coefficients to the parameters α_k and β_k of the model.

The derivation of this result is given in an Appendix at the end of the paper. There it is shown that $d_{T,j} = d_{T+1,j-1}$, $(j = 1,2,\ldots,M-T \;;\; T = 1,2,\ldots,M)$. Thus, it is necessary to obtain only $\{d_{T,0} : T = 1,2,\ldots,M\}$, and these are easily derived from the following matrix equation.

$$\begin{bmatrix} \alpha_p & \alpha_{p-1} & \alpha_{p-2} & \cdots & \\ & \alpha_p & \alpha_{p-1} & \cdots & \\ & & \alpha_p & & \\ & 0 & & \ddots & \\ & & & & \alpha_p \end{bmatrix} \begin{bmatrix} d_{1,0} \\ d_{2,0} \\ d_{3,0} \\ \vdots \\ d_{M,0} \end{bmatrix} = \begin{bmatrix} \beta_{p+1} \\ \beta_{p+2} \\ \beta_{p+3} \\ \vdots \\ \beta_q \end{bmatrix} \tag{3}$$

The triangular nature of these equations evidently makes them particularly easy to solve. This is especially true when we note that $M = q - p$ is very rarely large.

Box and Jenkins give some non-seasonal examples in their book and a seasonal one is given in McKenzie (1984a) expressing the well-known airline model forecasts in current level, gradient and seasonal factor form. We give one other brief example here to illustrate the evaluation of the d-coefficients. Consider

$$(1 - \phi B)(1 - B)X_t = (1 - \theta B^4)a_t \; .$$

Now, $M = q - p = 4 - 2 = 2$. Thus,

$$\begin{bmatrix} \phi & -(1+\phi) \\ 0 & \phi \end{bmatrix} \begin{bmatrix} d_{1,0} \\ d_{2,0} \end{bmatrix} = \begin{bmatrix} 0 \\ -\theta \end{bmatrix}$$

i.e. $d_{2,0} = -\theta/\phi$ and $d_{1,0} = -\theta(1+\phi)/\phi^2$.

The Integrated Form of the forecast is now

$$\hat{X}_t(1) = b_1^t + b_2^t\phi + d_{1,0}e_t + d_{2,0}e_{t-1} ,$$
$$\hat{X}_t(2) = b_1^t + b_2^t\phi^2 + d_{2,0}e_t ,$$
$$\hat{X}_t(T) = b_1^t + b_2^t\phi^T , \ T > 2 ,$$

where

$$\begin{bmatrix} b_1^t \\ b_2^t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \phi \end{bmatrix} \begin{bmatrix} b_1^{t-1} \\ b_2^{t-1} \end{bmatrix} + \begin{bmatrix} (1-\theta)/(1-\phi) \\ (\theta-\phi^2)/\phi^3(1-\phi) \end{bmatrix} e_t .$$

DISCUSSION

While it is certainly true that the Difference Equation form is the simplest mathematically, it is important to note that it need not be the most efficient computationally. The relative efficiency of the two forms depends on what lead times are to be forecast and how often. This aspect of ARIMA forecasting has already been noted by Godolphin (1975) and more recently by McKenzie (1984a). The two forms correspond to two distinct approaches. The Difference Equation procedures generate forecasts recursively for all lead times up to the maximum required. Use of the Integrated Form, on the other hand involves the calculation of a set of values which can be used to construct forecasts for any lead times.

As an illustration, consider the ARIMA(0,2,2) process. Box and Jenkins (1970,p146-) discuss forecasting this process in detail using both approaches, so we simply reproduce the forms here. The Integrated Form is considered first. It has two distinct parts:

i) Revision Equations

$$b_0^t = b_0^{t-1} + b_1^{t-1} + h_1 e_t$$

$$b_1^t = b_1^{t-1} + h_2 e_t \qquad (4)$$

(ii) Forecast Equation

$$\hat{X}_t(k) = b_o^t + kb_1^t \qquad k \geq 1 \tag{5}$$

There is a fixed computational investment in generating $\underline{b}_t = (b_o^t, b_1^t)$ before any forecast can be obtained. Once this is achieved, however, forecasts for any lead times are easily derived using equation (5). All the storage requirements relate to the revision equations (4), and here involve b_{t-1}, h_1, h_2 and e_t.

For the Difference Equation form, the forecasts are generated directly:

$$\hat{X}_t(1) = \hat{X}_{t-1}(2) + \psi_1 e_t$$

$$\hat{X}_t(2) = \hat{X}_{t-1}(3) + \psi_2 e_t \tag{6}$$

$$\hat{X}_t(k) = 2\hat{X}_t(k-1) - \hat{X}_t(k-2) \qquad k \geq 3 \tag{7}$$

Note that there are no revision equations which do not generate forecasts. However, it is clear that the forecasts of lead times k=1,2 and 3 play a role in equations (6) and (7) similar to that of $\underline{b}_t$ in equations (4) and (5). Another important point here is that to obtain forecasts for any lead time using (6) and (7) we must first generate the forecasts for all shorter lead times.

To illustrate the differences between the two approaches we consider some simple forecasting scenarios. Suppose we wish to forecast for lead times k = 1,2,...,T. The computational requirements are clearly comparable. The Integrated Form requires less storage to generate the forecasts, but the revision equations necessitate a little more arithmetic than the use of (6) and (7). Suppose now that the lead times of interest are not consecutive. For example, suppose we wish forecasts for lead times k=1,2,3,6,12 and 24. Using the Integrated Form, we need to generate only these six forecasts. [illegible]

$\underline{b}_t$ has been obtained. However, the Difference Equation form requires the generation of all 18 forecasts. Alternately, we may routinely forecast for lead times 1,2 and 3 and occasionally require forecasts for others, e.g. 6,12, 18 etc. Again, it is easier to generate these via (4),(5) than (6),(7).

Another prediction of common interest is the cumulative forecast, i.e. the forecast of $Y_t(T) = \sum_{k=1}^{T} X_{t+k}$. It is given by $\hat{Y}_t(T) = \sum_{k=1}^{T} \hat{X}_t(k)$. Using the Difference Equation form we must generate $\hat{X}_t(k)$ for $k=1,2,\ldots,T$ and then sum them. For the Integrated Form, we can use a single forecast:-

$$\hat{Y}_t(T) = \sum_{k=1}^{T} (b_0^t + kb_1^t) = Tb_0^t + \tfrac{1}{2}T(T+1)b_1^t$$

These comments apply equally well to other situations in which linear functions of future values are to be predicted. They are usually more easily treated using the Integrated Form.

There is no suggestion here that one approach is always better than the other. Rather that each has certain advantages in certain situations. The investment in the Revision Equations of the Integrated Form yields greater flexibility in forecast generation and a variety of forecasting problems are thus more efficiently treated by this approach. To assess the cost of such an investment, note that the dimension of the vector $\underline{b}_t$ is the order of the autoregressive-difference operator in the ARIMA process, e.g. (p+d) in an ARIMA(p,d,q) process. Thus, revision of $\underline{b}_t$ is roughly comparable to the production of forecasts for the first (p+d) lead times using the Difference Equation form. Clearly, if no lead times beyond (p+d) are to be predicted, the Difference Equation form is more efficient in general. However, when lead times in excess of (p+d) are required, the better approach depends, as we have seen, on a number of factors.

On the other hand, there is a most important advantage enjoyed by the Integrated form. It lies in the area of interpretation. This is an aspect of forecasting which should not be underestimated. We may recall Stern's comments (1974) that managers who require forecasts may be prepared to accept trends and seasonal effects because these correspond to familiar ideas. This view is merely one of many such cris de coeur still heard by forecasters. The true value of a forecast is not invested solely in its accuracy but also in its credibility. The former can be assessed only after the event forecast, whereas the latter will determine whether the forecast is used at all. The modelling of time-series as ARIMA processes is now a common practice, thanks to the availability of a variety of powerful computer packages. Neverthel- a major hurdle for most users is still the practical interpretation of the forecasts.

The reason that the Integrated form enjoys an advantage here is that it generates the forecast via the Eventual Forecast Function (EFF). This is a linear combination of deterministic functions of the lead time T. For fixed T, the functions are known exactly, but their coefficients in the linear combination adapt with each new observation. Thus, in the ARIMA(0,2,2) example, the EFF is given by equation (5). The deterministic functions are $f_1(T)=1$ and $f_2(T)=T$. Their coefficients, the components of $\underline{b}_t$, are revised with each new observation via equations (4). The new observations allow the forecast to adapt, while preserving the basic structure of the EFF.

This structure is most important because the individual deterministic functions which appear in the EFF can often be readily interpreted in terms of familiar concepts such as trends, growth and seasonality. It is interesting to note that it is the practice of the more ad hoc forecasting systems such as Holt-Winters or Exponential Smoothing to model the forecast as

just such a linear combination of deterministic functions. Surely , the success and wide acceptability of these systems is at least partly due to the fact that their predictions are cast in a familiar form. In addition, more recently popularized systems such as the Bayesian models of Harrison and Stephens (1976), or the Kalman Filtering approach discussed by Harvey (1983), are also constructed around these same familiar components of trend and seasonality, etc. On the other hand, none of the software available for modelling and forecasting ARIMA processes can generate forecasts in this readily understood form. Nevertheless, such interpretations are available via the components of the EFF in the ARIMA forecast. These components may be extracted, monitored, and projected.

As an illustration, we consider the monthly sales of US houses (January, 1965-December,1975). The data are given in Abraham and Ledolter (1983), and the fitted model there is $(1-B)(1-B^{12})X_t = (1-0.2B)(1-0.83B^{12})a_t$. The errors from this model were obtained from the data using backforecasting. The components of the Integrated Form of the forecast were evaluated and the necessary revision equations obtained using the procedures described in McKenzie (1984b). The revision equations are

$$
\begin{aligned}
b_0^t &= b_0^{t-1} + b_1^{t-1} + 0.7465e_t \\
b_1^t &= b_1^{t-1} + 0.0113e_t
\end{aligned}
\qquad (8)
$$

$$
\begin{aligned}
c_k^t &= c_{k+1}^{t-1} + (0.0595-0.0113k)e_t \qquad k=1,2,\ldots,11 \\
c_{12}^t &= c_1^{t-1} + 0.0935e_t
\end{aligned}
\qquad (9)
$$

The forecast equation is

$$\hat{X}_t(k) = b_0^t + kb_1^t + c_k^t \qquad (10)$$

At time t, when X_t has been observed and $e_t = X_t - \hat{X}_{t-1}(1)$ evaluated, the

revision equations yield current values for the non-seasonal component $\underline{b}_t = (b_0^t, b_1^t)$, and the seasonal component $\underline{S}_t = (S_1^t, S_2^t, \ldots, S_{12}^t)$. These components have useful and readily understood interpretations. In the non-seasonal component, b_0^t is the current level of the process, and b_1^t the current gradient of the linear trend, i.e. the predicted rate of change in level per month. In the seasonal component, S_k^t is the additive seasonal factor for month $(t+k)$. It predicts the amount by which the data will deviate from the process level k months from now (time t).

Exhibit 1 about here.

Exhibit 1 displays the data for the years 1968-75, and the corresponding values of the current level. The values of the gradient are not displayed, but follow a path similar in shape to that of the level, though with a different vertical scale. In January 1968, with a data value of 35, we find the current level is 41.8 and the gradient 0.067. Thus, in this month, the non-seasonal component of the forecast for T months ahead is given by (41.8 + 0.067T). Clearly, as the data evolved, this proved to be optimistic, as the trend during the next two years appears to be downwards. This is reflected in the January 1970 gradient 0.027, which is smaller, but still positive. From equation (6), we can see that the gradient adapts only slowly with each new observation. In January 1971 and 1973, the gradient adapts to the increasing level, with values 0.173 and 0.236 respectively. In the latter month, the predicted linear trend in the process T months ahead is (59.7 + 0.236T). As Exhibit 1 clearly shows, this would have been a very poor long term prediction. However, by January 1974, the predicted trend is (41.6 - 0.061T). This, in the event, may be a little pessimistic. It is important to note that not only have we been able to

quote the non-seasonal component without reference to the seasonal one but, from equations (8) and (9), we can calculate them separately. Thus, for example, we could forecast using the Difference Equation form and monitor the level and trend directly via (6).

Exhibit 2 about here

The values of the seasonal component are revised with every observation and can be plotted in the same way as the non-seasonal one. Since it involves twelve paths and they are, in fact, very stable, they are not reproduced here. It is just as revealing, in this case, to plot the current seasonal component of the data, i.e. $X_t - b_c^{T}$. This is displayed in Exhibit 2. There is a very clear and consistent seasonal pattern. The months March until August are always high and approximately the same. The other months are equally consistent, with the possible exception of December, which appears to be on a downward trend. It is interesting to note that when a subset of these data (up to May 1975) was analysed earlier by Hillmer and Tiao (1979) those authors chose a model in which the seasonal component was deterministic. The applicability of such a model is reflected in Exhibit 2. We would have much more confidence in predictions of the seasonal component, $\underline{S}_t$, of this forecast than the non-seasonal one, $\underline{b}_t$.

SUMMARY

The purpose of this paper is to make a case for the usefulness of the Integrated Form of an ARIMA forecast. A brief review of its derivation is presented. A new method for obtaining an essential part of it is given in detail. This form of the forecast has often been neglected in the literature but it has some useful characteristics. These derive from its direct use of the Eventual Forecast Function. In particular, we have emphasised two points.

First, it need not be, as is often supposed, computationally inferior to the Difference Equation approach. Secondly, it offers, via the EFF, considerable advantages for interpretation of the forecast. In both cases, the argument is illustrated with an example.

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APPENDIX

Derivation of the d-coefficients

First, note that substituting equation (2) into the usual forecast revision identity $\hat{X}_t(T) = \hat{X}_{t-1}(T+1) + \psi_T e_t$, and using the revision equation $\underline{b}_t = L\underline{b}_{t-1} + \underline{h}e_t$ yields $d_{T,j} = d_{T+1,j-1}$, $(j = 1,2,\ldots,M-T \; ; \; T = 1,2,\ldots,M)$. Thus, it is necessary to obtain only $\{d_{T,0} : T = 1,2,\ldots,M\}$ and this may be achieved as follows.

Applying the usual conditional expectation arguments to X_t to derive T-step ahead forecasts for $T = p+1, p+2,\ldots,q$, yields

$$\sum_{k=0}^{p} \alpha_k \hat{X}_t(p+r-k) = \sum_{i=0}^{M-r} \beta_{p+r+i} e_{t-i} \qquad (r = 1,2,\ldots,M) \tag{11}$$

Using equation (2) and the fact that $\sum_{k=0}^{p} \alpha_k f_i(k) = 0$, the left-hand side of (11) may be written as

$$\sum_{k=\omega}^{p} \alpha_k \{\hat{X}_t(p+r-k) - \sum_{i=1}^{p} b_i^t f_i(p+r-k)\}$$

where $\omega = \max(0, p+r-M)$. A further application of equation (2) yields

$$\sum_{k=\omega}^{p} \alpha_k \sum_{i=0}^{M-r+k-p} d_{p+r-k,i} e_{t-i} \tag{12}$$

which is to be equated to the right-hand side of (11).

Equating coefficients of e_{t-i} for $i = 0,1,\ldots,M-r$, we find

$$\sum_{k=\omega_i}^{p} \alpha_k d_{p+r-k,i} = \beta_{p+r+i} \tag{13}$$

where $\omega_i = \max(0, p+r-M+i)$, for $0 \le i \le M-r$, $1 \le r \le M$. From these equations (13) it is easy to derive the matrix equation (7).

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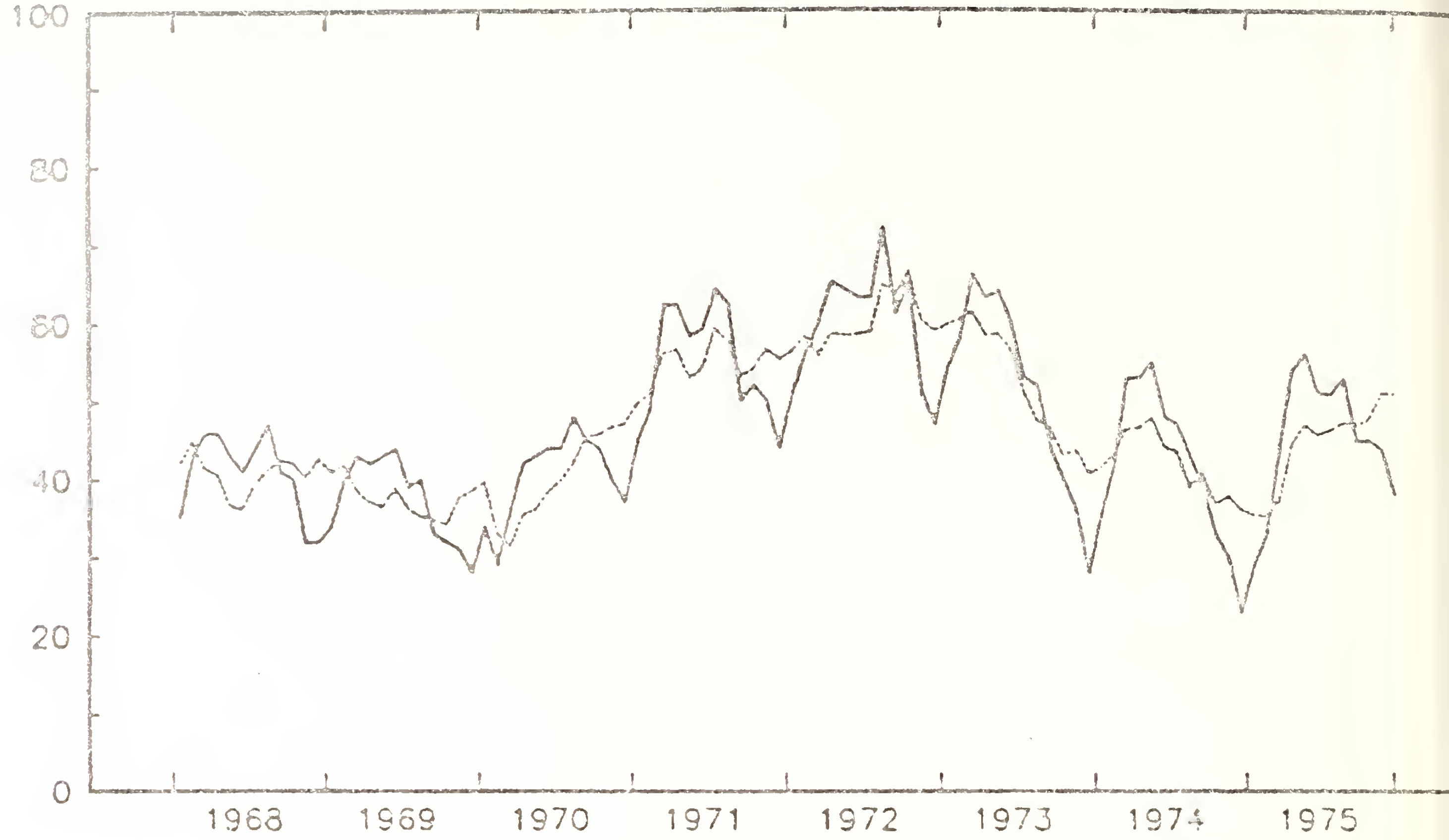

Exhibit 1. US House sales, Jan 68–Dec 75.
Solid line:Data; Dotted line:Current Level b_o^t

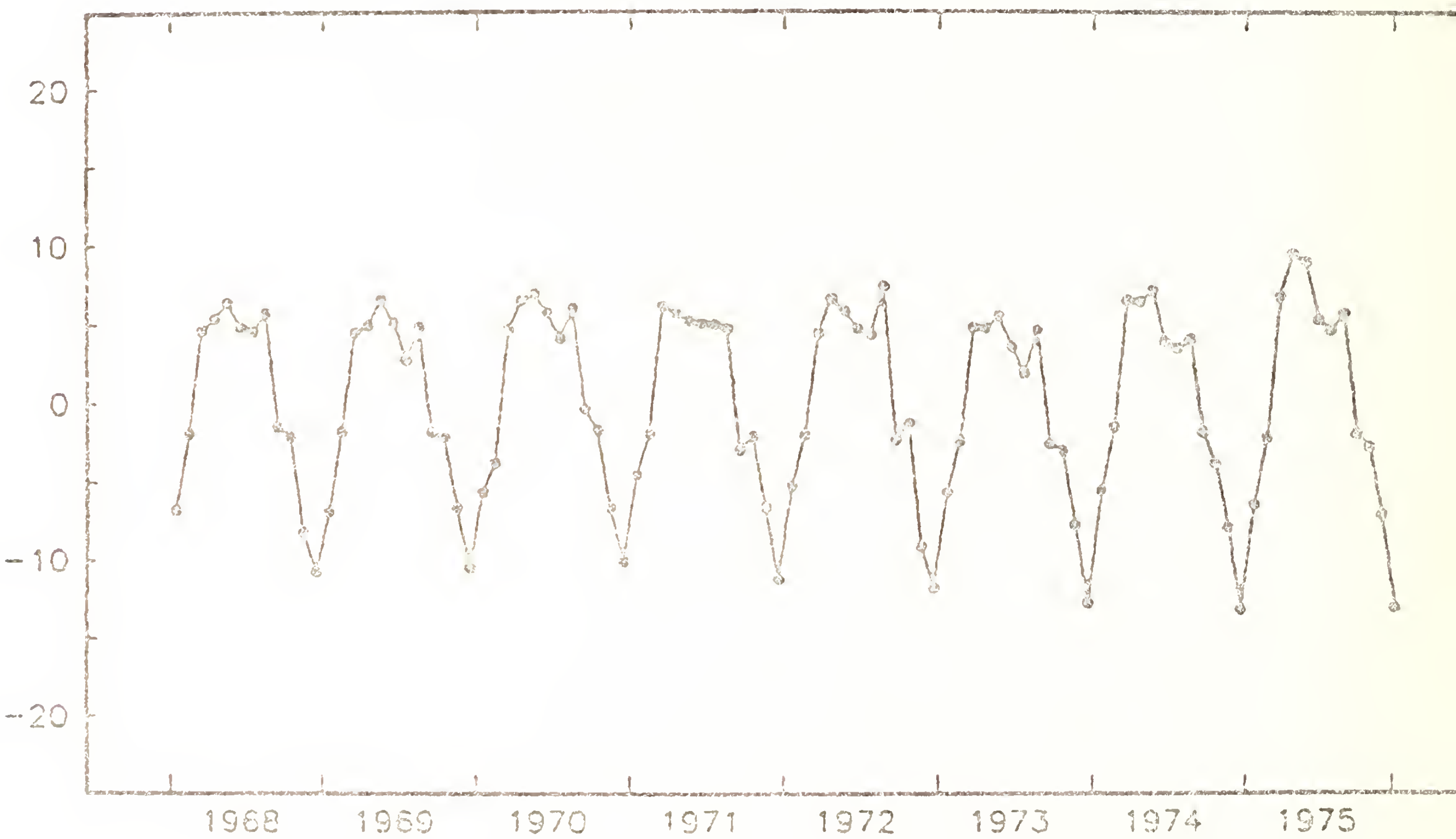

Exhibit 2. US House sales: residual seasonal pattern
after removal of Current Level b_o^t

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